ALLIGATION.

By Willard A. Ballou.

In consideration of the almost absolute assurance with which the authors of arithmetics have omitted the topic of alligation from their texts for the past generation or two, I will confess that it is with some timidity that I am undertaking to show that this old-fashioned, illogical process greatly simplifies many of the calculations of a special but a rapidly increasing class of men, namely the industrial chemists.

You will better appreciate my timidity when I quote some of the not altogether enthusiastic references to this process by eminent mathematicians. Dr. Fench back in the eighties, after a long lifetime of work as a mathematician and educator made this apologetic remark concerning alligation:

"Alligation medial is largely applied to business affairs. Alligation alternate is more a mathematical curiosity; still there is a limited class of problems that have no other arithmetical solution."

Personally I am inclined to think that there are some problems that may be solved by alligation that have no other practical solution. Dr. D. E. Smith's appreciation may be correctly judged by the following quotation:

"A person *may* have an exercise in logic by studying alligation—merely indeterminate equations in an awkward, mediæval form."

Alligation, although it is held in favor in Germany, is rapidly and happily (as one author states) disappearing from the American and English text books.

I am willing to venture that one of the reasons why it remains a favorite topic in the German schools is because of the use that the chemists of that country find for this process, although I have very little direct evidence on this point.

Since many of the younger generation are not familiar with the processes of alligation I may be tolerated to explain and illustrate the more important principles.

To begin with let me take the following simple problem:

A cubic inch of Cooper's gold weighs 0.405 lb. What is its composition?

Cooper's gold is an alloy of copper and platinum. If we take the specific gravity of platinum as 21.5 and of copper as 8.83, we shall find that one cubic inch of platinum weighs approximately 0.775 lb. and one cubic inch of copper 0.318 lb.

Algebraic Solution.

x = the volume of the copper in cubic inches present.
y = the volume of the platinum in cubic inches present.

$$
\begin{aligned}
.318x + .775y &= .405, \\
x + y &= 1.000, \\
.318(1 - y) + .775y &= .405, \\
.318 - .318y + .775y &= .405, \\
.457y &= .087, \\
y &= .1904.
\end{aligned}
$$

That is there is 19.04 per cent. of platinum in the alloy.
x will then equal .8096 or 80.96 per cent. of copper in the alloy.

By Alligation.

	1.	2.	3.	4.	5.
Pt		0.775	0.370	1/370	87
	0.405				
Cu		0.318	0.087	1/87	370
					457

$$\frac{457}{87} \times 100 = 19.04 \text{ per cent. Pt}, \quad \frac{370}{457} \times 100 = 80.96 \text{ Cu}.$$

In column 1 is the weight of the mixture.

In column 2 are the components or the simples.

In column 3 are the excess or lack in weight.

In column 4 are the medial ratios or the respective gains or losses referred to unity.

In column 5 are the quantities that should be mixed to give the required mixture, *i. e.*, 87 lbs. of a substance weighing 0.775 per unit volume when mixed with 370 lbs. of a substance weighing 0.318 lbs. per unit volume will give 457 lbs. of a mixture that will weigh 0.405 lb. per unit volume.

This of course readily checks, for:

$$\begin{aligned} 87 \times 0.775 &= 67.425 \\ 370 \times 0.318 &= 117.66 \\ \hline 457 \times 0.405 &= 185.085 \end{aligned}$$

In this example one cubic inch was taken to facilitate the computation although naturally in practice the volume of the sample would not be any integral quantity, however its volume is determined with sufficient accuracy by measuring its displacement.

This example illustrates for us the first three important principles of alligation and although self-evident they are worth while stating.

Principal One.—The required average must be greater than some of the components and less than others.

Principle Two.—To produce a compound of a given average the losses caused by those components below the average must be balanced by the gain on those components above the average.

Principle Three. Any column of medial ratios may be divided or multiplied by any number without affecting the required average.

Even in this simple form involving two variables it saves the chemist much labor in certain common calculations of frequent occurrence. Following is given the two ordinary solutions of a simple problem of "a mixture with a common constituent" and after that the same problem solved to the same degree of accuracy by the method of alligation. I do this in detail that you may compare the amount of work involved in each method of solution. As the problems grow in complexity the greater becomes the power of alligation over the other methods until we arrive at a type of problem that has no other practical solution of which I am aware.

The silver from 4.22 grams of a mixture of AgI and AgBr weighed 2.11 grams. What is the weight of the iodine and of the bromine present in the mixture?

$X =$ weight of AgI present. $Y =$ weight of AgBr present.

$$X + Y = 4.22,$$

$$\frac{107.88}{234.8} X + \frac{107.88}{187.8} Y = 2.11,$$

$$Y = 4.22 - X,$$
$$0.45945X + 0.57444Y = 2.11,$$
$$0.45945X + 0.57444\,(4.22 - X) = 2.11,$$
$$0.45945X + 2.42414 - 0.57444 = 2.11,$$
$$0.11499X = 0.31414,$$
$$X = 2.7319 \text{ grams of AgI,}$$

$$2.7319 \times 0.540545 = 1.4767 \text{ grams of I,}$$
$$4.22 - (2.11 + 1.4767) = 0.6333 \text{ grams of bromine.}$$

This is a perfectly clear, logical solution, well understood by the student but laborious in execution, especially when the problem involves the solving of a system of four or five equations.

Solution by Proportion.

$2.11 \times 2.176493 = 4.5924$ grams of AgI $\equiv 2.11$ grams of Ag. $4.5924 - 4.22 = 0.3724$ grams, deficiency of the mixture due to the lighter atomic weight of bromin and proportional to the amount of bromine present.

$$\frac{126.92 - 79.92}{79.92} = \frac{0.3724}{X},$$

$$X = \frac{79.92 \times 0.3724}{47} = 0.633324 \text{ grams of Br.}$$

$$4.22 - (2.11 + 0.63324) = 1.47676 \text{ grams of I.}$$

This is a very good, brief method of solution for this type of problem but one not readily comprehended by students and in cases not so simple as this illustration liable to prove confusing.

Solution by Alligation.

Ag in AgBr from table 57.444 per cent.
Ag in AgI from table 45.945 per cent.
Ag in mixture (problem) 50.000 per cent.

AgBr	50.000	57.444	7.444	$\frac{1}{7444}$	4055	$\frac{4055}{114.99} = 35.264$ per cent. AgBr
AgI		45.945	4.055	$\frac{1}{4055}$	$\frac{7444}{11499}$	$\frac{7444}{114.99} = 64.736$ per cent. AgI

$64.736 \times 4.22 = 2.73185$ grams of AgI.
$2.73185 \times 0.54054 = 1.4767$ grams of I.
$4.22 - (2.11 + 1.4767) = 0.633$ grams of Br.

It should not be passed over without notice that this method leaves the 35.264 per cent. of AgBr free to be used as a check, for the checking of this class of computation is of the utmost importance. By the other methods the check is as laborious, if not more so, than the solution itself.

It is desired to mix manganese ores containing 23 per cent., 41 per cent., and 47 per cent. of manganese respectively, so as to obtain an ore containing 39 per cent. of manganese.

1	2	3	4	5	6	7	8	9	10	11	12
39	23	16	$\frac{1}{16}$	$\frac{1}{16}$	$\frac{1}{16}$	8	2	10	1	2	3
	41	2	$\frac{1}{2}$		$\frac{1}{2}$		16	16		16	16
	47	8	$\frac{1}{8}$	$\frac{1}{8}$		16		16	2		2
								42			21

Column 9 gives us the required mixture namely 23.8 per cent. of 23 per cent. ore, 38.1 per cent. of 41 per cent. ore, and 38.1 per cent. of 47 per cent. ore; but suppose the supply of 47 per cent. ore is limited so that it is not desirable to use so high a percentage of that ore. Divide column 7, the column containing the medial ratio in integral form which involves the 47 per cent. ore, by 8, writing the result as column 10; reproduce column 8 as column 11; and then column 12 will give another mixture which will produce a 39 per cent. ore, but one containing only 9.52 per cent. of the 47 per cent. ore. Both of these mixtures check, as well as an infinite number of other mixtures that might be found in the same way to conform to almost any limitations likely to be imposed by the commercial world. It is in this property that the power of alligation lies when it is applied to commercial problems.

From the above class of examples three other principles of alligation may be deduced:

Principle Four.—An original medial ratio in alligation alternate consists of two terms only.

Principle Five.—The sum of two or more medial ratios is itself a medial ratio.

Principle Six.—A medial ratio derived from other medial ratios by addition consists of a number of terms not less than two nor greater than the number of simples or components.

Samples of different lots of coal showed on analysis sulphur 2.4 per cent., 1.2 per cent., 1.7 per cent., 2.4 per cent., and 1.1 per cent.

Since coal containing more than 1.5 per cent. of sulphur forms excessive slag and burns out grate bars, it is desired to know in what percentage these different grades of coal shall be mixed so that they shall not average more than 1.5 per cent. of sulphur.

	1.1	4	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$				2	9	12				23	34.33 per cent.
	1.2	3	$\frac{1}{3}$				$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{3}$				2	9	12	23	34.33 per cent.
1.5																	
	1.7	2	$\frac{1}{2}$	$\frac{1}{2}$			$\frac{1}{2}$			4			3			7	10.45 per cent.
	2.4	9	$\frac{1}{9}$		$\frac{1}{9}$			$\frac{1}{9}$			4			3		7	10.45 per cent.
	2.7	12	$\frac{1}{12}$			$\frac{1}{12}$			$\frac{1}{12}$			4			3	7	10.45 per cent.
																67	

This problem is very practical and shows how the medial ratios of each gain must be compared with the medial ratios of each loss when there are several of each, and how the derived medial ratios in integral form are combined by addition to give the proper quantities to be mixed.

As to the extent that the chemists may make use of the process I would mention the mixing the different normals to obtain the one desired, also in the dilution of chemicals in solution to any desired strength. It is very useful in mixing compound alloys, *i. e.*, in obtaining an alloy of desired composition from other alloys whose composition is known. In mixing alcohols or other liquids that show a contraction you cannot make use of the process without correcting for this contraction by the use of tables giving this contraction. However, you can easily change the specific gravity to the corresponding percentage by weight and then proceed without regard to the volume occupied by the mixture. I have often used it to advantage in the dilution to the desired strength of chemicals in solution. This is analogous to the only practical use I have found alligation put to in this country and that by the wholesale drug houses. They tell me that they could scarcely do business without it.

An order comes in for .78 morphine and they have in stock only .725 and .803 morphine. They work out the quantities by simple alligation and mix to the desired strength.

I do not know that I would advise the teaching of alligation to all classes of students, but I would strongly advise it for all people going into certain lines of practical work and especially for chemists. It would help them out of many of their difficulties and would facilitate certain operations that they now perform by tedious methods.

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